# Teacher's speech and gesture as a communicative and strategic

# tool to convey and discuss mathematical concepts in a bilingual

# Algebra classroom

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# Abstract

Educational researchers and the mathematics education community have recently begun to focus on the importance of students engaging in disciplinary conversations. Additionally, researchers' interest in designing more effective learning opportunities for students whose first national language is not English, have begun to investigate the important differences between conversational English and academic English. However, research has shown that academic English is difficult to master and even native English speakers struggle in conveying mathematical reasoning using the proper registers. A challenge that English Language Learners (ELLs) encounter is learning academic content and language simultaneously. Given the importance of academic English to learning and assessment, we need to better understand how teachers can influence students' discourse practices in ways that lead students to bridge their existing and often informal mathematical ideas to the formal registers of mathematics. This paper will assert that gesture (combined with speech) was a tool that a bilingual teacher used effectively to convey and discuss algebraic concepts with Hispanic ELL high school students. We argue that gesture is an important visual resource that can play a valuable role for ELLs in creating effective discourse practices and environments.

# **Introduction (1)**

Educational researchers and the mathematics educational community have recently begun to focus on the importance of students engaging in disciplinary conversations (Forman, 1996; Lampert, 1989; Lampert, Rittenhouse, & Crumbaugh, 1996; McClain & Cobb, 1998; McNair, 1998; O'Connor, 1998; O'Connor & Michaels, 1996; 1993). Studies show that students can learn mathematics at a deep level of understanding when creating an effective set of classroom discourse practices that lend in mathematical communication and academic English. The push for mathematical discussion and academic English is critical given that students are assessed based on their understanding of written texts and their ability to communicate mathematical concepts in writing and at times via discussion.

Research studies have also shown that academic English is difficult to master. Even native English speakers struggle to interpret and communicate in mathematical registers<sup>1</sup> (Pimm, 1987). The challenge is even more complicated for ELLs who must learn academic content and language simultaneously. For this reason, researchers' interest has focused on designing more effective learning opportunities that investigate and pinpoint the important differences between conversational English and academic English for the growing population of students whose first national language is not English.

Given the importance of academic English to learning and assessment, we need to better understand how teachers can influence students' discourse practices in ways that lead students to bridge their existing and often informal mathematical ideas to the formal register of mathematics. Teachers' implementation of discourse practices in their classrooms will depend on the way they conceptualize the relationship between mathematical discourse and understanding. Based on these perceptions, they will likely attend to different aspects of the classroom and utilize a different set of

<sup>&</sup>lt;sup>1</sup> Register refers to the specialized use of terminology that applies within a particular community of practice such as the mathematics community.

pedagogical practices. For ELLs, how teachers model mathematical discourse and utilize mathematical registers is particularly important.

By the start of the next century, Latinos will become the largest minority group in the United States (United States Department of Census, 2001). These demographics will affect both K-12 and higher educational systems. To date, schools that are not equipped with appropriate or sufficient resources have underserved Latino students. Pedagogical practices that are not effective in fostering academic English development and academic success have failed to address the needs of these Spanish-speaking ELLs. The traditional question, "Why do 'they' (ELLs) fail?" has been scrutinized by educational researchers in hopes of reducing the number of students who persistently fail in U.S. schools. However, research has yet to resolve how to close the "achievement gap" or how best to facilitate the transition from everyday language to academic language<sup>2</sup>.

Research on monolinguals shows that participating in discourse practices can be a tool that facilitates students' learning academic content as well as acquiring the appropriate skills needed to communicate academically. This research also argues that scholarly debate and argumentation can promote higher levels of thinking as students present their own ideas and then justify or defend those stated positions (Lampert, 1989a; 1989b). Yet some teachers struggle to incorporate these practices in their monolingual and bilingual classrooms because of the limited support available to them in making informed pedagogical decisions about how best to implement discourse practices.

One resource that has yet to receive much attention but has the potential to help students participate in academic conversations where they are simultaneously learning mathematics and academic English is gesture. Gesture, combined with speech and graphical resources, is a communicative tool that is readily available to all teachers and students and can be used to support meaningful conversations. In addition to being ubiquitous, gesture is a communicative resource that

<sup>&</sup>lt;sup>2</sup>For example, in mathematics, the word *leg* represents a measurement of a right triangle whereas in everyday English, *leg* represents a body part.

is shared across cultures and national languages. It is worth exploring the extent to which gesture and speech in a mathematics classroom facilitates a teacher and students' discussion of mathematical concepts without their being restricted to mathematical registers or to academic English.

In this paper, we will briefly summarize the research literature on gesture as a communicative tool and the perspectives and controversies surrounding ELLs discourse practices in academic contexts. Second, we will provide the general background of this study and outline the methods for a case study in which a Hispanic teacher in an urban high school conveyed and discussed mathematical concepts to his bilingual students. Third, we will present our findings that address the following research questions:

# How does a teacher use talk and gesture to convey mathematical knowledge? How does a teacher use talk and gesture to increase students' participation?

We will argue that the unification of the teacher's gesture, in conjunction with speech and graphic resources, helped students grasp and discuss mathematical concepts. Our results will show that the teacher's gestures and talk assisted in clarifying, explaining, highlighting, and emphasizing mathematical concepts to his students. Additionally, gesture and talk resolved multiple meanings, elicited students' justification of their thinking, and advanced the mathematical lesson. We will conclude the paper by discussing how this case suggests that more attention be paid to helping students transition from participating in mathematical conversations to participating in these conversations in a normative fashion, marked by the discipline specific vocabulary and conversational norms. We will use our findings to raise additional questions about how the role of gesture may need to change as students come to understand mathematical concepts and transition to the formal register of mathematics.

# **Theoretical Framework (2)**

Historically, gestures have been used as part of communication. Whether pointing to show direction(s), requesting the time by tapping on the wrist, nodding the head to indicate "yes" or "no," or shrugging the shoulders to indicate "I don't know," gestures involve forms of physical action, serve as visual representations or enactments of objects, and accompany, replace, align, or mismatch with speech.

Researchers have identified gesture as a communicative resource. Moreover, studies show that students have used gesture and talk to illustrate, convey, or explain their reasoning. Gesture and the use of supplemental entities, such as visual artifacts, enable students to communicate prior to their initiation of a standard discourse or prior to their access to the discipline's terminology (Church & Goldin-Meadow, 1986; Gullberg, 1997; Kendon, 1997; Roth, 2001; Roth & Welzel, 2001.) The use of gesture and speech to communicate not only assists students but also teachers, who have used gesture and speech in the classroom in order to express themselves better without having to rely on speech alone. Some teachers have used gesture to support their talk in conveying information of a specific academic discipline, especially when referencing difficult, general, or abstract academic topics (Corts & Pollio, 1999).

The use of speech and gesture not only facilitates communication but also helps some people cognitively. Whether a person is searching for a word or communicating in a different language, gestural movements and utterances provide an avenue to articulate thoughts, hunches and hypotheses without relying on speech to explore an idea. Research shows that gesture as a cognitive function enables the speaker to: explore knowledge (Hanks 1992); recall more details (Goldin-Meadow, 1999); create mental representations (Koschmann & LeBaron 2002; Roth & Lawless 2002); organize spatial information for verbalization (Alabli, 1999); retain concepts (Alibali & Goldin-Meadow, 1993); and construct inter-subjectivity (Koschmann & LeBaron, 2002). Whether gesture is used as a tool to clarify imprecise thoughts or as a tool to explore thoughts that may be difficult to think through in a verbal format, the role of gesture as a cognitive resource can influence the learning environment. Although gesture can assist in reducing cognitive burden, gesture as a cognitive tool is beyond the scope of this paper. Relevant to this paper, gesture and speech serve two main functions. The first function is as a tool to *communicate,* where gesture and speech can be used to convey and discuss academic concepts. The second function is as a tool for English Language Learners to engage in *mathematical discourse practice*.

### **Gesture as a Communicative Tool (2.1)**

Gesture has numerous roles depending on the context, the listener(s), and the speaker who uses them. Kendon (1997) outlines six specific types of gestural categories that are used in different conversational situations. For all of these categories, body movement and facial expressions are the main gestural performances that convey and/or interpret talk. First, gesture and speech are part of a social interaction wherein the speaker and the listener(s) regulate each other's attention so that talk is sustained. It requires them to participate by being attuned to each other's talk and gesture. Second, listener's gestures assess the speaker's utterances. More specifically, a body movement gesture, such as nodding the head or a facial expression of approval, acceptance, understanding or negation is an indication of assessing the talk in progress. Third, gesture is part of the conversation as a deictic reference. For instance, a speaker can shift from an auditory to a visual focus in order to convey a topic, a concept, or an idea. Fourth, gesture alters the meaning of speech so that the gesture is the focal point of the talk. For example, a speaker's talk could be comprehensible ("I am having a good day"), but with a gesture (rolling the eyes), the talk has a different meaning. Fifth, gesture guides the speaker's next turn so that talk transitions from the speaker to the listener. In other words, the listener can interpret a gesture as a segue to becoming the speaker in the discourse.

Sixth, gesture is used to understand when spoken utterance is missing. In this case, gesture is an alternative to speech.

Of Kendon's six categories, only four<sup>3</sup> are relevant to our analysis and will be discussed in this paper. These four are: gestures can be used to assess someone else's utterances and aid in an iterative process of establishing intersubjectivity; deictic gestures can be used to direct attention to a physical space in order to create a domain of scrutiny (Goodwin, 1986) by emphasizing parts of the environment that are relevant or are being referred to; iconic and metaphoric gestures can be used to redundantly express or elaborate an idea; and gestures can be used in place of a word altogether. Although some of the studies mentioned focus on everyday social interactions, they are are still broadly applicable to educational contexts.

# Gesture for assessing or grasping utterances (2.1.1)

Listeners can use gesture to indicate their assessment or comprehension of a speaker's talk by making inferences about an activity, contributing to an activity, or projecting future events. For instance, if a listener frowns, nods, or holds up a hand, the speaker will interpret the gesture and possibly adjust how he communicates to the listener (DeFornel, 1992; Goodwin & Goodwin, 1992). A listener can also elicit talk and gesture simultaneously or after a speaker has produced a gesture; this is referred to as a return gesture (DeFornel, 1992). A *return gesture* is visible and reflexive movement that replies to the speaker's ongoing talk. It is produced with talk and with an action performed in a preceding sequential context. A *return gesture* acknowledges the verbal expression and interprets the iconic contribution. It also displays the listener as an active participant who analyzes and participates in the talk. It becomes a resourceful tool that enables people to connect the [iconic] gesture, inference, and context (DeFornell, 1992). The use of speech and gesture as a

<sup>&</sup>lt;sup>3</sup> The other two classifications *regulating patterns of attention* and *speaker's next turn* are significant in the study of gesture but are not pertinent to this paper. For more information on *regulating patterns of attention* see Heath (1992) and Goodwin & Goodwin (1986). For more information on *guiding the speaker's next turn* see Streeck & Hartge (1992).

collaborative and interactive activity is a process of assessment or understanding that requires participation from listeners (Goodwin & Goodwin, 1992).

Gesture can be used to indicate understanding of a speaker's utterance. Although the term gesture covers a wide range of actions, from an iconic gesture to a simple gaze, regardless of the gesture a listener produces, a speaker is able to interpret it as some form of assessment as the talk is in progress. Most teachers and students use gesture and speech in order to understand each other's utterances. Some teachers use their students' gestures and utterances to understand students' talk (Roth & Lawless, 2002). Some students use gesture and speech to express themselves in articulating their understanding of a concept (Church & Goldin-Meadow, 1986; Goldin-Meadow, 1999). Some students' gestures are indications of "readiness to learn" (Church & Goldin-Meadow, 1986; Goldin-Meadow, 1986; Goldin-Meadow, 1999; McNeill, 2001). In sum, gesture and speech is a resourceful tool that students use to convey their thoughts or reasoning of a concept.

### Gesture used as a deictic reference (2.1.2)

The use of gesture and talk can serve for different purposes. Some people use gesture and talk simultaneously where gesture is not only part of the talk but also important in order to understand the speaker (Goodwin, 1986; LeBaron & Streek, 2000). Without the gesture, the talk would become incomprehensible. For example, LeBaron and Streeck (2000) focus on a college professor who uses gestures as a public performance to critique and evaluate students' architectural model design. According to LeBaron and Streeck, the professor's deictic gesture (pointing) highlighted certain aspects of a model as a method to draw attention to a specific feature or section of it. Gesture enabled the professor to convey information as he shifted between gesture and talk simultaneously. Moreover, the use of gesture facilitated in transforming objects to symbolic action, comprised cognitive and communicative functions, and was needed in order to understand the specialty of the content. Instructors are not the only individuals in the educational systems that use

representational gesture and talk to convey meaning. Research studies (Alibali, 1999; Goldin-Meadow, 1999; Kelly & Church, 1998; Roth & Lawless, 2002) show that students use representational gesture and talk to express their thoughts.

Deictic gesture can serve for at least two other functions (Goodwin, 1986). The first function is determining the significance of a gesture that accompanies the talk. The speaker has the option to use gesture as the main target of the talk (e.g. "Why is this here?") or secondary to the talk. The second function makes gesture salient. In other words, the speaker could show the gesture before verbalizing or referencing an object. The gesture becomes a preview to the talk (e.g., pointing while saying, "I left on the chair."). Regardless of how a speaker uses a deictic gesture, it serves for three purposes (Goodwin, 1986). First, gesture enables the speaker to transition from a verbal to a visual mode. Second, gesture is not only restricted to specific events but also emerges, changes, and disappears as talk changes. Third, the listener is attuned to the gesture so that the talk is comprehensible. Gesture and speech is not only a resourceful tool to communicate with the listener(s), but is also a purposeful method that facilitates learning in an educational setting.

#### Gesture used to alter speech (2.1.3)

Having a conversation requires share knowledge of the talk as well as a speaker eloquently conveying an idea or thought so that listener(s) are able to comprehend the talk. The speaker determines (consciously and subconsciously) when to use gesture and what role it will play in in the talk. For instance, in the educational context, when a concept is new or abstract, some speakers use gesture to align their talk in order for discourse to evolve and become established.

Roth (2001) focused on high school students in an introductory physics course who used *Interactive Physics* and communicative gesture to facilitate their learning. Through the use of speech and gestures, students discussed concepts of physics without having a formal introduction to the study of motion, expressed the relationship between object and trajectory, force, and velocity

before adopting the formal language of physics, and maintained an interaction with each other as their usage of the terminology in physics increased. The students aligned their talk and gestures so that a discourse could evolve and be established. The listeners, in this case the teacher and classmates, had to bridge the talk and gesture so that the talk was comprehensible.

Research studies with graduate students,<sup>4</sup> who are familiar with medical terminology, show similar results where gesture supported or replaced talk in order to convey knowledge (Koschmann & LeBaron, 2002). Moreover, gesture and speech assisted in articulating and finishing each other's talk (inter-subjectivity). Gestures provided students with the means for conveying their knowledge without being restricted to specialized registers. This also seems to be the case for undergraduates discussing mathematics (Edwards, 2003).

Studies show that gestures are used by students with diverse educational levels (Edwards, 2003; Koschmann & LeBaron, 2002; Roth, 2001). Gestures are not only restricted to learning new concepts, but they also helped students who were familiar with a concept to accentuate their knowledge and comprehension of a topic. For instance, Koschmann and LeBaron's (2002) study on second-year medical students found that students used gesture and talk to convey their knowledge and understanding of a specific content discussion to their professor. In these two studies, gesture and accompanying talk played different roles – an earlier role and a later role. The earlier role was experimental whereas the later role was communicative. The studies show that in the educational system, gesture and talk are imperative for a speaker to elicit a discourse or convey a thought, idea or knowledge without the specificity of the registers.

For some students, classroom discourse can be a challenge, specifically in mathematics classes. Popular prejudices about learning mathematics have reinforced the belief that mathematics is difficult to understand, discuss, learn, and master. Some students have internalized these societal

<sup>&</sup>lt;sup>4</sup> First and second year medical students as well as graduate nursing students.

beliefs as a justification for their limited mathematical skills and knowledge. Other students have been socialized to believe that their lack of fluency in English hinders them in learning mathematics, specifically in solving word problems. Although there is no guarantee or recipe that will engage students in their learning, teachers are constantly implementing various pedagogical practices in their classrooms in hopes of teaching the academic discipline conceptually. In short, the use of gesture and speech has been found to enable students to communicate with their peers and teacher.

### Gesture as a substitute for talk (2.1.4)

For some people, verbally communicating thoughts or ideas using speech only can be difficult. Some people (consciously and subconsciously) use gesture to support or substitute talk. Gesture can enable listeners understanding when talk is absent or limited. For example, Corts and Pollio (1999) focus on how a college professor used gestural performance and speech to convey abstract psychological concepts to his class. Three college lectures on abnormal psychology where the professor used gesture were analyzed to show the relationship between spontaneous use of figurative language and nonverbal gestures. The professor in the study used gesture to replace (and supplement) his talk. Gestures and metaphors served to present physical representations that guided students. Corts and Pollio's study is essential because it suggests that gesture was purposeful. Professors as well as teachers can use gesture as an additional resource for students to grapple with the topic or content especially when the concepts are new or abstract.

Teachers have used gestures to provide students with another alternative for learning (Alibali, Flevares, & Goldin-Meadow, 1997; Corts & Pollio, 1999; Kelly & Church, 1998). Gesture can enable students to communicate thoughts, reasoning, or hypothesis of an academic topic without restricting them to academic terminology. For some students who are learning terminology or the English language, gesture and speech could be an additional resource to convey their thoughts to their peers or teachers. Therefore, it is essential to focus on the usage of gesture and speech in order to understand how teachers and students, specifically ELLs, communicate about a concept, thought, or an idea without having to rely on academic English.

# **English Language Learners Discourse Practices (2.2)**

Traditional frameworks on ELLs' mathematical learning and language acquisition have emphasized three perspectives: *acquiring vocabulary, multiple meanings, and participating in mathematical discourse*. Moschkovich (2002) critiques the common assumption that ELLs first need to acquire mathematical terms in English before they can engage with mathematical concepts. Moschkovich argues that mathematical discourse can be a resource for ELLs learning mathematics and English. In this section, the three traditional perspectives will be summarized. This section will then draw on Moschkovich's critique and highlight that gesture as a strategic pedagogical tool can enable ELLs to communicate mathematically.

Traditional ideas about the ways ELLs learn mathematics stress the importance of acquiring vocabulary first. This perspective defines learning as computation and solving traditional word problems, and it emphasizes vocabulary acquisition as a central obstacle for ELLs (Dale & Cuevas, 1987). However, knowing mathematical terminology does not encompass knowledge or understanding of mathematics (Moschkovich, 2002). In many U.S. classrooms, students are expected to participate in pedagogical practices that go beyond computation and word problems.

Although grammatical competence is important and may potentially be problematic to ELLs when they are expected to communicate mathematically (Moschkovich, 2002; Olivares, 1997), it should not be the main goal of mathematical learning. ELLs can learn the lexicon and the structure of the English language as they participate in mathematical communication. Although there is limited mathematic educational research on ELL high school students communicating mathematically, such inquiry is needed.

Traditional frameworks also stress that acquiring vocabulary is needed in order to clarify multiple meanings that exist in conversational and academic English. The claim is that multiple meaning of words can create obstacles for ELLs who are employing two national languages at the same time; therefore, according to traditional frameworks, clarification and construction of multiple meanings is the second perspective for ELLs learning mathematics in relation to English.

Mathematical learning requires that students learn to speak like mathematicians and acquire mathematical registers<sup>5</sup> (Pimm, 1987). Once students develop mathematical registers, meaning will be available in the language. However, in most mathematics classes, a mixture of registers occurs and failure to distinguish between the registers (ordinary and mathematical English) can result in miscommunication<sup>6</sup> (ibid). The creation of mathematics registers in terms of the role of metaphor *(extra-mathematical metaphor<sup>7</sup>* and *structural metaphor<sup>8</sup>)* is essential in developing mathematical registers and in understanding the processes involved in mathematical talk (ibid).

Studies show that monolingual students have difficulties with mathematical registers as well as with reading and understanding mathematical word problems (Pimm, 1987). The probability of ELLs encountering similar effects with the registers is even higher. ELLs must contend with registers that differ in everyday English, mathematical registers in two national languages, and translations between languages; therefore encountering twelve different possible sources of confusion as opposed to one for monolinguals (see Figure 1).

<sup>&</sup>lt;sup>5</sup> Pimm defines *register* as "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings" (p. 75). Pimm defines *mathematical register*, "the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is used for mathematical purposes…" (p. 76).

<sup>&</sup>lt;sup>6</sup> *Register confusion* is "the processes by which pupils attempt to assign meaning to the phrases and expressions they hear in mathematics classes are completely consonant with those by which they acquired and manipulated meanings as young children 'learning how to mean'" (p. 88).

<sup>&</sup>lt;sup>7</sup> To use *extra-mathematical metaphor* is to "explain or interpret mathematical ideas and processes in term of real-world events."

<sup>&</sup>lt;sup>8</sup> Structural metaphor is "an extension of ideas from within mathematics itself."



Of the numerous possible points of transfer confusion for ELLs, the most problematic are those that involve transfer with academic language. The first possible point of confusion is when ELLs make a transfer from mathematical talk in English to everyday talk in English. For instance, in mathematical talk "leg" represents a side of a right triangle whereas in everyday talk, it represents a body part. Second, ELLs make transfers from everyday talk in English to mathematical talk in Spanish. For example, the everyday English phrase "to have a concern [about something]" translates to *"tener un pendiente"* in Spanish; yet the word *pendiente* represents "slope" in Spanish mathematical talk. Third, ELLs may have difficulties transferring from mathematical talk in English to mathematical talk in Spanish. For instance, the English mnemonic device "FOIL" represents the procedure for multiplying polynomials. This device does not transfer well to Spanish, which does not have the same word. Fourth, ELLs must transfer from everyday talk in Spanish to mathematical talk in English. For example, in everyday talk in Spanish, *"un cuarto*" represents "a room" whereas in mathematical talk, it represents "one fourth." This example in red doesn't work well Fifth, ELLs transfer from mathematical talk in English to everyday talk in Spanish. For instance, in mathematical talk, "less," "less than," and "is less than," are three distinct phrases that have a different meaning whereas in everyday talk in Spanish, those three phrases have similar representation. Although five possible confusions have been listed, other possible confusion can occur. For ELLs, the absence of the right word may cause them to struggle to understand and convey mathematical knowledge to their peers and teacher.

Vocabulary can be complicated when ELLs are trying to transfer terms from everyday English which have different meanings in mathematics (Olivares, 1997). For instance, synonyms may present difficulties; ELLs may not know all of the mathematical terms that represent one mathematical word. If a word is introduced in English, they may not know it in their primary language. If they are familiar with the terminology, ELLs must then transfer the term from their primary language to mathematical English. In addition, the complexity where ELLs must recognize the *representative function*<sup>9</sup> and the *directive functions*<sup>10</sup> can be confusing. The distinction between these two functions is how a verb is used in a mathematical statement (Olivares, 1997; Pimm, 1987). Lack of understanding of a verb can result in misunderstanding of a mathematical statement and miscommunication between a teacher and students. These difficulties complicate ELLs learning to bridge everyday speech with mathematical representation.

The first and second traditional perspectives outlined above (*vocabulary acquisition* and *multiple meanings*) are limited and have the risk of being deficiency models for ELLs (Moschkovich, 2002). Instead of focusing on students' difficulties or limitations, an appropriate model would emphasize students' acquired abilities. Teachers should use multiple resources to communicate mathematically and expand on how ELLs learn mathematics. For instance, multiple

<sup>&</sup>lt;sup>9</sup> Descriptive information needed to solve a problem

<sup>&</sup>lt;sup>10</sup> Descriptive information needed to answer a problem

meaning can be a problem for students when they misinterpret a word or phrase that is in the academic register. However, it is important to note that while resolving multiple meanings across national languages and technical registers requires explicit attention to the discourse environment, it is not a deficit. Instead, the process for resolving multiple meanings can be a rich resource for meaning making (Moschkovich, 2002). Although educational researchers have not yet explored how to implement meaning making in bilingual classrooms, such inquiry is needed. such studies would provide a better understanding of ways to implement mathematical discourse as a pedagogical practice and ways students can learn mathematics and develop formal mathematical registers and academic English.

### ELLs participation in mathematical discourse (2.2.1)

The use of mathematical discourse practice in the classroom draws on a socio-cultural and situated view of learning and serves four purposes (Moschkovich, 2002). Moschkovich points out that using mathematical discourse in classrooms allows researchers to describe and elaborate on the variety of resources that ELLs use to communicate mathematically. Second, the identification of mathematical discourse practices could assist teachers in building-upon existing resources that ELLs can and may use in the classroom. Third, the scope of assessing ELLs of their competency in communicating mathematics broadens. Fourth, an intentional focus on mathematical discourse practices avoids building a deficiency model of ELLs.

Mathematical discourse can serve as a resource for ELLs as they enhance their listening and speaking skills in English as well as increase their mathematical knowledge. Students can bridge mathematical meaning and English when teachers: are aware of which concepts used in the classroom are unfamiliar to students (Garrison & Mora, 1999), build on students' prior knowledge and primary language (Moschkovich, 2000), and use students' native language as a resource (Guttierez, 2002). In addition, teachers should allow their students to work in collaborative groups

so that mathematics is discussed in two national languages, if necessary, as students solve problems and explain their reasoning using teacher-designed materials (Gutierrez, 2002). Furthermore, teachers should consider the importance of the cultural information of terms and context used in the classroom so that frustration and miscommunication is minimized (Secada, 1992; Olivares, 1997; Gutierrez, 2002).

Scholarly literature recommends possible pedagogical strategies and practices that could facilitate in ELLs' learning mathematics. Traditional frameworks have emphasized that learning vocabulary is the initial step needed so that students can experience optimal learning of mathematics through discourse. However, Pimm and Olivares highlight the possible challenges students might encounter with mathematical registers and functions. Moschkovich (2000; 2002) argues that an exclusive focus on acquiring vocabulary can potentially result in deficiency models whereas an emphasis on mathematical discourse can potentially serve as a positive resource for students.

Through the use of gesture and talk, students can discuss topics (and terminology) without a formal introduction (Roth & Lawless, 2002). Moreover, research shows that teachers can use gesture to help their students understand their intention(s) without an emphasis on terminology (LeBaron & Streeck, 2000). Students will then be able to discuss academic concepts that can be new, difficult, or abstract. Additionally, gesture as a visual resource becomes a tool that can assist in resolving register confusion. For instance, Heath (1992) shows how a doctor communicated with his patient without relying on medical registers. Through a series of nods from the patient, the doctor was able to ascertain that the patient understood her medical directions. Finally, the use of gesture and talk is an important resource for constructing meaning (Alabali, 1999; Alibali et al., 1997). Students can construct successful interactions between their classmates and teacher which facilitates participation and mathematical discourse. Gesture and speech can serve to interpret knowledge,

making the usage of gesture a resource to elicit and enhance discourse and mathematical knowledge in particular for ELLs who struggle in bridging two national languages.

# Methods (3)

# Case Background (3.1)

The data used in our study was obtained from a videotape corpus from a larger study of an urban high school mathematics department in a Midwestern city in the U.S. The student population of the school was predominately Hispanic (58.8%) and on an average were at the lower end of the socio-economic status (SES) continuum; 65% were eligible for free or reduced lunch (Roy, Bohl, & Rousseau, 1998). Our case study focuses on a bilingual mathematics class where the students' English language proficiency ranged from limited to proficient in two languages (English and Spanish).

### Participants (3.2)

This study focused on a single teacher, Mr. Garcia (all names are pseudonyms), a Latino male in his second-year of teaching. Mr. Garcia is a bilingual teacher who learned Spanish as an adult. Although Mr. Garcia teaches different levels of Algebra courses, this study focused on his bilingual Algebra class where the majority of the students were repeating the course.

The instructional content that Mr. Garcia used in his classroom was taken from *Connected Mathematics*, a curriculum that was designed for middle school students (Fey, Fitzgerald, Friel, Lappan, & Phillips, 1998). Mr. Garcia was oriented to producing artifacts that could be included in students' portfolios since the primary evidence for learning at this school was portfolios. Our study analyzed a mathematical assignment to investigate the relationship between two hypothetical runners, Tara and Ingrid, who began a race at different starting points and ran at two different constant speeds. Students were instructed to first predict the winner and then to justify their reasoning by writing a paragraph. They were then expected to individually present their reasoning

by graphing their linear equations to support their predictions. Students used their data to interpolate and extrapolate predictions and to create a graphical representation (linear equations) of the two runners. The teacher and the students then publicly discussed their predictions, methods, and reasoning in a whole group setting.

# **Data Collection (3.3)**

The data source for our analysis is the video recordings of the two class days over which the race word problem spanned (approximately two hours total). The videotapes were completely transcribed, and segments of talk when the teacher gestured were marked and recorded on the transcripts. These segments were then analyzed qualitatively for the types of gestures that were used and the role the gestures played in facilitating the construction of shared meaning between the teacher and students.

#### Data Analysis (3.4)

Before documenting any of the teacher's gestures, we watched the videotapes of the classroom and of interviews with the teacher and read the transcripts a few times. Then, using the classroom videotapes, one videotape at a time, all of the teacher's gestures were time indexed for starting time and duration which required consistently pausing and rewatching segments of the videotapes numerous times. Next, a detailed description of each of the teacher's gestures and the accompanying speech were documented. Given that gestures occur very quickly, we rewatched the videotapes after the documentation to assure that all of the teacher's gestures were documented and the detailed descriptions matched the gestural episode and the time frame.

The gestural episodes that did not reference mathematical instruction or talk were omitted from the transcripts. For example, Mr. Garcia instructed his students where not to sit by using his finger to point at a group of chairs. This and other similar gestural episodes were not included in the analysis. We were liberal in terms of what was included as "mathematics" in order to make sure that all relevant mathematical talk between the teacher and his students was included. For instance, there were some episodes during which it was difficult to hear a student's talk (on the videotape) but Mr. Garcia, who apparently heard his students, elaborated, clarified, or restated the talk using speech and gesture. Gestural episodes such as this one were counted as instances of mathematical talk.

The videotapes were employed to analyze how speech and gesture were interchangeably used in order for the teacher to convey mathematical concepts and mathematical discussion with his students in a whole-group setting. Kendon's (1997) six categories on how gesture serves as a communicative tool were used to categorize the gestures. Moreover, the gestures were also categorized based on the language the teacher used prior, during, or after the gestural performance.

Pertinent excerpts will be illustrated in the results section, which has been divided into two different subheadings: *how a teacher used talk and gesture to convey mathematical knowledge* and *how a teacher used talk and gesture to increase students' participation*. It is important to note that some excerpts could be encompassed in other subheadings. The purpose of the excerpts is to show and explain how speech and gesture consisted of various functions based on the environment, objects and setting. Denoting the different functions talk and gesture played is vital in order to understand and comprehend how a teacher's speech and gesture facilitated mathematical teaching in a bilingual setting.

# **Results (4)**

The results will show how gesture and the accompanying talk served as a communicative tool that enabled Mr. Garcia and his students to discuss mathematical concepts, procedures, reasoning, and hypotheses. With Mr. Garcia's effective use of gesture, students' lack of familiarity with mathematical registers and their limited English fluency did not restrict them from having a discourse in mathematics.

The teacher's gestural performance and talk are discussed in two main sections. The first section, *teacher's talk and gesture for conveying mathematical knowledge*, represents the teacher's speech and gesture that was used in the classroom to explain, elucidate, highlight, and emphasize his talk about a mathematical concept. This section also includes his use of gestures to maximize the English language yet minimize potential mathematical predicaments, such as misconceptions or errors, in order to create shared vocabulary and inter-subjectivity. The second section, *teacher's talk and gesture to increase student participation*, focuses on the utilization of the teacher's speech and gesture to understand, acknowledge, or emphasize students' talk, engaging them in mathematical discussions. Both of the sections will include detailed examples of the teacher's use of these kinds of gestures as he discussed with his students the relationship between two runners, Tara and Ingrid, who began from different starting points and ran at constant speeds. Since students had already predicted the winner, justified their rationale in a written paragraph, and presented their reasoning by graphing their linear equations, the examples in this section are taken from the class discussions of students' predictions, methods, and reasoning.

# Teacher's talk and gesture for conveying mathematical knowledge (4.1)

#### **Explanations and clarifications (4.1.1)**

A tactic that Mr. Garcia used in his class to express himself was to gesture while speaking. In the following excerpt (Excerpt 1), students discussed who won the race. In order to determine the winner of the race, students had to first identify which line represented Tara and which represented Ingrid. Mr. Garcia asked his class, "Who was winning the race?" A student responded, but the response was incorrect. Mr. Garcia then used gesture, which was aligned with his speech, as an additional resource for students so that they could create meaning and understanding in an alternative mode.

1	Т	So before the forty seconds- or before	
2		the four seconds- Who's ahead before	
3		four seconds? Antes de los cuatro	
4		segundos, quién está enfrente, quién	
5		está adelante? [Before the four	
6		seconds, who is ahead, who is in-	
7		front?]	
8	SN	Nobody ( ).	
9	Т	No- not at the point but before.	Teacher extends his hands as his right index finger touches his left pinky finger.
10		At two seconds, for example, who's	
11		ahead?	
12	S	Tara.	
13	SN	Tara. Tara.	
14	Т	Tara is ahead. And then at six	
15		seconds who's ahead?	
16	SS	Ingrid.	
17	Т	Ingrid. So before this time- now I	Teacher points to one of the two lines.
18		lost the two names again. Tara, we	Teacher points to one of the two
19		said And after this time Ingrid	lines.
20		is ahead. Okay. Very nice.	
Excerpt 1: 1	Explana	ations & Clarifications	

Excerpt 1. Explanations & Clarineations

In this excerpt<sup>11</sup>, Mr. Garcia asked his students in English and then translated into Spanish which runner would be ahead after four seconds (lines 1-7). A student responded in English, "Nobody" which was incorrect. Mr. Garcia used gesture to clarify the question (line 9). He extended his hands as his right index finger touched his left pinky finger to denote two lines intersecting. In other words, instead of saying, *not at the point of intersection*, he used his hands to illustrate that the point he was referring to was not the intersecting point. The teacher's pinky and his index finger became a visual performance and an additional resource. Immediately after Mr. Garcia used this speech and gesture together, students responded correctly (lines 12 and 13). Mr.

<sup>&</sup>lt;sup>11</sup> **BOLD** text represents gesture

<sup>...</sup> represents time/pause

<sup>()</sup> represents talk that the audio recording did not capture

<sup>[]</sup> represents a translation

S, SN, SS represent different student's voices

correctly (line 16). With the use of speech and gesture, Mr. Garcia was able to moderate the classroom discussion so that students were able to answer his questions.

In line 17, Mr. Garcia restated his students' utterance as he pointed to one of the two lines that were before the intersecting point, which was projected on the board. He then repeated his action of pointing, but this time the line he was pointing to was the point of intersection (lines 19 & 20). Mr. Garcia's gesture and talk made the relevant lines visually salient to students, which was crucial in order to determine who was ahead in the race before a certain time (the intersecting point) and who won the race.

Mr. Garcia's gesture served as a performance where he built meaning so that students could map verbal and visual information. For example, in Excerpt 1, Mr. Garcia's gesture and talk reformatted his question so that students understood the question and answered correctly. In this case, gestures helped to elicit positive responses from students, thereby encouraging them to actively participate in class discourse. Mr. Garcia's gesture assisted in eliciting students' positive responses to participate in the class discussion.

# Highlighting or emphasizing (4.1.2)

Further into the same lesson on Tara and Ingrid's race, Mr. Garcia used gesture to align with his own and students' talk. These gestures highlighted and emphasized a mathematical concept. In the sequence of events in Excerpt 2, students were expected to find the rate of change for each of the runners. After doing so, students compared the two runners' rate of change to make an appropriate mathematical conclusion regarding who won the race. The excerpt begins where Mr. Garcia asked a series of questions. When a student spoke in Spanish, the teacher used gesture to highlight the student's talk. The teacher then restated and highlighted the student's rationale as the teacher aligned his gesture with talk.

1 2 3 4 5 6	SN	Que Tara- Tara recorre diez metros por segundo y que Ingrid recorre veinte metros por cada segundo. [That Tara- Tara covers ten meters per second and that Ingrid covers twenty meters per second 1	Teacher points to the x-axis and then moves his hand to the line that represents Tara. Teacher points to the x-axis and then moves his hand to the line that represents Ingrid
7	Т	Very nice Velocity or speed Let's check	mai represento ingita.
8	-	that out. How are you getting that,	
9		Bernardo? How can you tell by looking	
10		at the graph?	
11	S	Si Tara empieza en el cuarenta, entonces	
12		está en cero, no?	
13		[If Tara starts at forty, then she is at zero,	
14		right?]	
15	T	Okay. Uh-huh.	
16	S	Entonces, un segundo pasa y llega a	Teacher points and moves his
17		cincuenta. Dos segundo y llega a	finger on the line.
18		sesenta. Tres, a setenta, y asi.	
19		[So, one second passes and she gets to	
20		sixty. Three, to sevently, and so on 1	
$\frac{21}{22}$		sixty. Three, to seventy, and so on.]	
22	т	Very nice Make a little triangle like	Teacher moves his finger in order
23	1	this	to create a right triangle
25		He's saving that <b>in one second she goes</b>	Teacher moves his finger
26		from forty meters to fifty meters. Ten	horizontally to represent time and
27		meters. So in one second she goes ten	vertically to represent distance.
28		meters. En un segundo nada mas pasa de	····· · · · · · · · · · · · · · · · ·
29		cuarenta a cincuenta, son diez metros.	
30		What's another name for what you just	
31		gave me, Bernardo? Or anybody else?	
32		Speed? Velocity? What's the other	Ticks off fingers with each
33		word for that? The word that we use	different term.
34		in this class probably every day?	
35			
36	S	Rate of change.	
37	Т	Thank you. Rate of change. We just	Teacher points.
38		found the rate of change. And you also	
39		went and gave us the rate of change for	
40		Ingrid, also.	

40 Ingrid, a Excerpt 2: Highlighting Talk



Mr. Garcia's gesture paralleled a student's talk. As the student communicated his mathematical ideas verbally, Mr. Garcia pointed to the graph (lines 1-4 and 17-19). When the student made the claim that "Tara ran ten meters per second" (lines 1-2), Mr. Garcia pointed to the x-axis and then moved his hand to the line that represented Tara. The student then made a claim that "Ingrid ran twenty meters per second" (lines 3-4), and Mr. Garcia pointed to the x-axis and then moved his hand to the line that represented Ingrid. When Mr. Garcia pointed to the x-axis and pointed to the line that represented Tara, his gesture visually reinforced to his students the representation of the x-axis and the line.

Mr. Garcia continued to align talk and gesture when he restated in English his student's Spanish utterance as he moved his finger to create a right triangle (lines 23-24). By using gesture to represent a right triangle, Mr. Garcia highlighted the concept of the rate of change as he moved his finger horizontally to represent time and vertically to represent distance in meters. Additionally, Mr. Garcia showed his students how the proportion was obtained for both runners, Tara and Ingrid (lines 25-28). In other words, rather than asking students to find the rate of change, he used gesture and speech to illustrate the concept. He then connected the visual aspect (right triangle gesture) and referenced it to the mathematical terminology as he mapped the terminology to the runners' trajectory (line 37).

Excerpt 2 illustrated that the student was not told how to obtain the answer; instead, Mr. Garcia's gesture highlighted his student's response as a logical process. The teacher's gesture assisted in the logical process. The teacher's gesture emphasized how the students obtained the answer, which helped the other students to follow and understand their classmate's talk. Moreover, the teacher's gesture may have been a resource for the student as the student structured his response. The gesture also served as a mechanism for Mr. Garcia to check for his student's understanding of a mathematical concept. Mr. Garcia's utilization of gesture was purposeful for illustrating the rate of change and bridging the concept with the term. The gesture and accompanying speech also helped Mr. Garcia to restate his student's reasoning, to reformulate the talk, and to reinforce the mathematical register.

The excerpt shows that Mr. Garcia's gesture was used to assess his student's understanding of rate of change, to stress a particular mathematical concept (line 37), and to guide students with the concept and the terminology. Overall, Mr. Garcia's gesture and talk served as a communicative tool that enabled a discussion on the concept of rate of change.

### **Projecting the English language (4.1.3)**

Aside from teaching math, one of Mr. Garcia's responsibilities was to teach English. One task was to teach content and another was to develop students' English proficiency so that they could graduate from high school. Mr. Garcia emphasized the English language by using numerous techniques. One of the techniques that he used was code-switching and gesture. By code-switching and simultaneously using gesture, Mr. Garcia was not only using speech to connect the talk from one language to another but also using gestures as a visual tool to reinforce the English language. Another technique that Mr. Garcia used in his class was restating students' Spanish utterances in English and using gesture. By restating his students' utterances, he modeled academic English as well as mathematical registers in order to assist students in adopting appropriate terminology and syntax.

In Excerpt 2 a student explained his rationale in Spanish (lines 1-4), and Mr. Garcia accentuated the appropriate mathematical terminology in English language for what the student had said (line 7). The teacher then posed additional questions that required the student to focus on the graphical representation to elaborate and justify his rationale (lines 8-10). The student responded in Spanish; however, the mathematical registers were not part of the student's talk (lines 11-22). Mr. Garcia restated his student's rationale as he used gestures, mathematical register, and English (lines

23-34). The teacher probed his students to identify a particular mathematical phrase, rate of change, which described what the student had conveyed to him (lines 34-35). In line 36, the student responded with the correct mathematical terminology.

Mr. Garcia's gesture and accompanying speech served as a tool to reinforce the concept of rate of change as he referenced the mathematical terminology in English. Moreover, Mr. Garcia illustrated to the class that an appropriate justification was required. The teacher's gesture and speech lent in continuing to speak in English and in asking his student another question, to which the student responded in English (line 36). In general, Mr. Garcia's gesture and speech provided a visual image of the student's talk so that other students could bridge the English with the gesture; otherwise, some students might have encountered difficulties understanding Mr. Garcia since he spoke in English and used mathematical terminology.

#### **Resolving multiple meanings (4.1.4)**

Bridging from everyday English to mathematical registers is an ongoing challenge for students learning and understanding mathematics in a second language. The lack of certain vocabulary in English can lead ELLs to struggle to understand and convey mathematical knowledge to their peers and teachers. In Mr. Garcia's class, the use of gesture and speech enabled students to participate in class discussions of mathematical concepts as they drew on their primary language and their mathematical abilities.

In Excerpt 3, a student expressed her mathematical reasoning about Tara and Ingrid's race in Spanish. A phrase that the student used revealed a case where multiple meanings needed to be resolved in order to solve the problem. Mr. Garcia used his gestures to help resolve the issue and to minimize potential confusion among other students.

- 1 S En que Tara llegó... salió más pronto que-
- 2 que Ingrid. [That Tara got there ... she left
- 3 before Ingrid.]
- 4 S Tara *empezó* desde los cuarenta metros y

5		esta Ingrid empezó desde abajo, desde el	
6		principio. [Tara started at forty meters and	
7		Ingrid started from the bottom, from the	
8		beginning].	
9	Т	Okay. Now, when you say, " Salió más	
10		pronto", I agree with the second part you	
11		said- she started forty meters ahead,	
12		whereas Ingrid started at zero. Qué	
13		significa, "Salió más pronto"? [What does	
14		" Salió más pronto" mean?]	
15	SN	( ).	
16	Т	Huh?	
17	SS	( ).	
18	Т	I don't know. "Más pronto" es igual que	
19		adelante o enfrente? [Is " más pronto" the	
20		same as in front or ahead?]	
21	S	No.	
22	Т	So, " más pronto", como por ejemplo, sea	
23		[So, " más pronto", for example, is-]	
24	SN	( ) antes ( ). [( ) before ( ).]	
25	Т	Antes. [Before.]	
26	SS	Antes. [Before.]	
27	Т	So, for example, you're saying,	Teacher moves his right and left
28		if Ingrid is here and Tara is here,	hand up and down. He then points
29		than Tara has started the race and then	in front of him but does not say
30		so many seconds later Ingrid started the	anything.
31		race. Ah. Okay. Interesting. Does anyone	Teacher walks forward.
32		else see that differently?	Teacher swings his arm.
1 7	· N/I1		

**Excerpt 3: Multiple Meaning** 



This excerpt begins with a student conveying to Mr. Garcia her rationale of the race in Spanish (lines 1-8). Mr. Garcia then asked for clarification of a specific phrase, *más pronto*, which she had used in Spanish that was not clear to him (lines 12-13). Although there was some response from the students (lines 15 and 17), Mr. Garcia pressed his students to explain what *más pronto* meant (lines 18-20). It was important for Mr. Garcia to identify what the student meant because the multiple meanings of *más pronto* may have referenced speed (who ran more quickly) or time (who

left earlier). In other words, depending on how the student was using the phrase, three different rationales could justify why a runner won the race. The runner may have won the race because she ran faster, because she started to run the race before the other runner began, or because she had less total distance to travel. These three rationales are clearly different (although not mutually exclusive), and clarifying the interpretation of the word was necessary.

Mr. Garcia asked his students in Spanish if *más pronto* was the same as "in front" or "ahead" (lines 18-20). While Mr. Garcia was working to resolve the multiple meaning with his students, in line 24, a new student engaged in the discussion to clarify the phrase. Immediately after Mr. Garcia repeated the word, *antes*, (line 25), a group of students' voices echoed the same word simultaneously (line 26). Rather than assuming that the entire class understood that *antes* represented time, Mr. Garcia used gesture and speech to animate the clarification of the word and to restate his student's rationale of why Tara won the race.

In line 28, building on this apparent convergence between the teacher and students, Mr. Garcia moved his hands as if someone was standing on either side of him. He then pointed forward, but did not say anything. By pointing, he was indicating an imaginary finish line was front of him. He then walked forward as he simultaneously said, "That Tara has started the race" (line 29). He then took a few steps forward, animating the role of Tara. Next, Mr. Garcia swung his arm and simultaneously said, "And then so many seconds later Ingrid started the race" (lines 30 & 31). By moving his arm, Mr. Garcia demonstrated that the second runner, Ingrid, had started the race.

The above excerpt shows that Mr. Garcia's acting out "time" and not "speed" created a visual image for students to grapple with the context-specific meaning of the phrase, *más pronto*. Mr. Garcia spent time to resolve these different meanings so that students would not misinterpret the student's rationale. If Mr. Garcia had, on the other hand, just corrected the student, he may have misunderstood how she was thinking about the problem and may have discouraged her and others

from further participation. This excerpt illustrates how Mr. Garcia's talk and gesture helped resolve multiple meanings by drawing on the full set of resources available to his students. His gestural performance enabled him to convey his student's rationale about who had won the race. In addition, the teacher paired his gesture with mathematical terminology to make sure he understood what his student was saying, at the same time creating an environment where his students could communicate their knowledge and strategies without being restricted to formal academic English.

In this section of the paper, we have shown how Mr. Garcia's gesture and speech played a role in highlighting key elements, sharing logical reasoning, replacing gesture and linking it to the register, and providing visual alternatives in order to help students resolve multiple meanings. These findings are important for four reasons. First, students can misinterpret a teacher's explanation of a mathematical concept and develop mathematical misconceptions. Second, gesture can be used as a visual tool that can lend in stressing mathematical concepts. Third, gesture can clarify terminology that has multiple meanings so that students can continue to discuss mathematical concepts with a common ground of shared vocabulary.

# **Teacher's gesture to increase student participation (4.2)**

Teaching mathematics requires various pedagogical practices including listening to student talk. For teachers, students' verbal expressions as they communicate mathematically can provide snapshots of understanding. Moreover, student talk becomes a tool that provides a teacher with direction for changing pedagogical practices so that students can grasp the mathematical concepts. Although a thorough analysis of the student talk in the mathematical classroom is outside the scope of this paper, the following section documents how Mr. Garcia's speech and gesture were used to understand and restate his students' talk.

This section of the paper focuses on how Mr. Garcia's gesture and speech were used to understand, acknowledge, or emphasize students' talk. Excerpts demonstrate how gesture was used as a mechanism to help the class in meaning-making. The data also shows how gesture was an initial step in encouraging students to participate in class discussion. Mr. Garcia's gestures are categorized as either *encouraging students to justify their thinking* or *using students' contributions to advance the mathematical lesson*. For both of these categories, pertinent examples illustrate the use of gesture and talk to understand student talk, recognize student contribution, and use student contributions to extend the lesson.

# Encouraging students to justify their thinking (4.2.1)

Learning mathematics and discussing mathematics in a whole group setting using a second language may be difficult for some ELLs. Teachers have continuously struggled in conveying mathematical knowledge in multilingual classrooms whose students range in their mathematical and language abilities. In addition, encouraging students to support or contradict each other's mathematical responses can be a challenge for some teachers (Pimm, 1987). Implementing discourse practices where students discuss each other's contributions can be a possibility when a teacher utilizes gesture and speech.

In Excerpt 4, Mr. Garcia wanted his students to understand that both runners, Tara and Ingrid, had run at constant speeds. In order for this to occur, students had to justify how a constant speed is determined, based on the given information. The teacher asked questions so that his students could discuss each other's mathematical reasoning. The excerpt illustrates how Mr. Garcia used talk and gesture to initiate a mathematical discussion among students where one student expanded on mathematical reasoning that was previously contributed by another student.

1	Т	Why do you say that Tara's keeping a constant speed? How can you tell that	
2		from the granh?	
3		from the graph?	
4	S	She ()- she's gone from a straight line.	
5	Т	Straight line.	
6		Is this a straight line? Is that	Teacher goes the board and, with his
7		straight? Sylvia says that means	hand and arm, follows the line.
8		She's going at a constant speed.	Teacher moves his hand.
9	SN	( ).	

10	Т	Ber- Ber- Bernardo's saying that if you	
11		look carefully at Ingrid's, it's also a	Teacher goes the board and, with his
12		straight line.	hand and arm, follows the line.
13		Are you thinking that right here it	Teacher cups his hand and moves his
14		starts to go slower?	hand up and down.
15	S	Yes ( ).	
16	Т	If you look at it really, really carefully,	
17		Sylvia. And if- when I read your paper,	
18		I looked at what I had up there again to	
19		check and see if maybe I bent the line	Teacher points.
20		there. But if you look carefully, it	
21		stays straight. It stays straight. But	
22		that's an excellent observation.	Teacher moves his hand in an upward
23		Constant speed. We can tell this	direction.
24		because the line is straight.	
25		Excellent, Sylvia. But we do need to	
26		also add here that Ingrid kept a constant	
27		speed as well	

# **Excerpt 4: Eliciting Justification**

This excerpt illustrates how Mr. Garcia's talk and gesture was used to elicit a discussion. For instance, the first student, S, stated that one of the runners, Tara, had a constant speed because the line representing her was straight (line 4). Mr. Garcia went to the board and, with his arm and hand, followed one of the two lines that were on the board and asked his students if the line was straight (lines 6-7). Mr. Garcia's gesture clarified which line they were referring to. He also emphasized his student's remark about the line being straight to the class. He then asked a question to the class as he bridged the mathematical terminology, constant speed and straight line, so that students would be able to discuss their classmate's rationale, either by agreeing or disagreeing. In other words, if students had not responded to the initial question because of the confusion of the terminology (constant speed and straight line), the teacher's gesture may have served as a clarification to reference the line and connect the terminology.

Repeating the same gesture a second time proved equally effective. Immediately after Mr. Garcia gestured and restated the student's claim (lines 5-8) that the straight line represented a constant speed, another student spoke (line 9). Although the audiotape did not capture the other student's talk, the videotape showed that Mr. Garcia listened to his student's talk. This assertion is

supported by Mr. Garcia's use of gesture and talk to restate this student's contribution (lines 10-12). Mr. Garcia mimicked the same gestural performance of moving his hand along a line on the board, but this time the gesture referenced the other line. Then, Mr. Garcia used the student's contribution that the other line, the line that represented Ingrid, was also straight in order to question the first student who had spoken (lines 13-14). As Mr. Garcia asked her a question, he pointed to a specific point on the line that represented Tara. Shortly after the student responded (line 15), Mr. Garcia continued to talk and used gesture to emphasize a point (lines 19-20) and constant speed (line 23).

In this excerpt, Mr. Garcia's gesture and accompanying talk not only served to illustrate the line and the point that he was referring to so that his students respond appropriately, it also served two other functions. One of the functions was to clarify a student's misconception of a line bending at a certain point. Mr. Garcia's gesture and talk served as an illustration that both lines were straight, thereby showing the runners' speeds to be constant. The other function was that gesture served to reinforce a second student's contribution that Ingrid's line was also straight and her speed was constant. Mr. Garcia's gesture and talk was purposeful in assisting students to juxtapose or elaborate on their mathematical rationale.

# Using student's contributions to advance the mathematical lesson (4.2.2)

Bridging students' contributions and mathematical concepts can be difficult for some teachers. In this section, a pertinent excerpt illustrates how Mr. Garcia used a student's contribution to advance the mathematical lesson. Through the use of gesture, Mr. Garcia was able to restate his student's talk and used his student's talk to elaborate on an explanation so that students would grasp a mathematical concept.

In Excerpt 5, the lesson was to determine who won the race by comparing Tara and Ingrid's running times. In order to obtain this conclusion, the class took four steps. The students first identified the representation of the *x*-axis and *y*-axis, identified which line represented each runner,

compared the times for both runners, and concluded that the runner who ran less time represented the winner of the race. When a student offered the answer "Porque lo hizo en menos tiempo" (Because she did it in less time) to Mr. Garcia's earlier question, Mr. Garcia was able to point-out the various steps that were needed in order to determine who won the race.

1	S	Porque lo hizo en menos tiempo	
2		[Because she did it in less time.]	
3	Т	Menos tiempo [less time]	Teacher quickly nods his head up and
4		You see that, Rose Marie?	down
5	S	Y Ingride se ( ).	
6		[And Ingrid was ()].	
7	Т	This is the time down here.	Teacher points to the <i>x</i> -axis of the graph
8		There's Tara's time.	Teacher draws two vertical lines and then
			points to the line.
9		It's much shorter.	Teacher uses his index finger and thumb
			to illustrate a gap.
10		If you run the race in shorter	
11		time, you must win.	
12	S	And Ingrid started to run slow, so	
13		they started to curve.	
14	Т	Very nice.	
	A 1	• • • •	

**Excerpt 5: Advancing the lesson** 

The first gestural performance that Mr. Garcia illustrated was a quick nod as he simultaneously said, "menos tiempo" (less time) in line 3. He then pointed to the *x*-axis to emphasize that it represented the time. Once he had established that the *x*-axis denotes time, he then drew two vertical lines to represent Tara and Ingrid. He was then able to bridge the *x*-axis (time) and the vertical line that represented Tara so that he could use his index finger and thumb to denote the space between the runners. He then stated the appropriate conclusion to be drawn from the difference in time in line 4 "*If you run the race in shorter time, you must win*."

The work of these four gestures, which elaborated and re-displayed the students' logic, lent in reviewing the mathematical concepts (e.g., analyzing the graph, comparing the lines on the graph and concluding with a reasonable justification), practicing a potential logic process that can be used to analyze and discuss future graphical representations, and advancing the lesson. Rather than just verbally elaborating the student's contribution by stating, "You run the race in a shorter time, you must win," Mr. Garcia's gesture visually depicted the four necessary steps for solving the problem. At the same time, he validated the student's response (line 14) and continued with the next topic in his teaching agenda. In other words, Mr. Garcia used the student's talk to fill-in the missing pieces of the mathematical reasoning process.

In this section, two excerpts illustrated how a teacher's gesture and speech were used as a strategic tool to initiate and increase students' discourse. Gesture and accompanying speech initiated a discussion among students rather than a discussion between a teacher and a student with other students listening. Through the use of gesture, the discussion was organized for other students to participate. This is important because how Mr. Garcia facilitated class discussions could either assist or hinder his students' participation in discussing mathematical reasoning. Moreover, in the excerpts, talk and gesture extended the mathematical lesson by reflecting or referencing students' contributions. This was vital because Mr. Garcia was recognizing his students' talk as a high caliber contribution that took the mathematical lesson to another level, reinforcing to the students that they are mathematical scholars. Creating an environment where the utilization of gestures was acceptable in order to understand and maintain mathematical communication was important in this classroom. Additionally, these excerpts demonstrated that displaying and/or restating students' discussing and learning mathematics.

The way that Mr. Garcia and his students worked through an algebraic word problem demonstrated how gesture and speech was usefully employed to clarify, explain, highlight, and understand a new mathematical concept. Gestural performances became a tool when students encountered verbal obstacles (e.g., limited English proficiency and/or limited disciplinary terminology) as they connected their prior knowledge with the new knowledge. As gestures complemented speech in the classroom discourse, students were able to construct and establish the meaning of a new concept. These results demonstrate that gesture plays a mediating role and that learning and knowing is tied to communication (Church & Goldin-Meadow, 1986; Goldin-Meadow, 1999; Roth & Welzel, 2001).

# **Discussion (5)**

In the class discussion about Tara and Ingrid's race, Mr. Garcia's gesture in conjunction with speech and graphic resources paralleled created meaning when language was abstract, general, or unclear to students. Moreover, the teacher's gestural performances became an additional strategic tool that lent in resolving multiple meanings, clarifying or reinforcing mathematical concepts, and advancing the mathematical lesson. The teacher was able to advance the mathematical lesson by highlighting and emphasizing particular aspects of his talk or the graphic displays, de-emphasizing the need for academic English language and the formal mathematical register when students were struggling to understand him or express themselves, and emphasizing and facilitating the use of English on the public floor. Gesture and the accompanying talk served as a communicative tool that enabled a teacher and his students to discuss mathematical concepts, procedures, reasoning, and hypotheses. Most importantly, students limited mathematical registers and English fluency did not restrict them from engaging in a mathematical discourse.

Mr. Garcia's gestural performance and talk were purposeful in making mathematical ideas meaningful to students, especially for new or abstract ideas like velocity. His use of gesture also facilitated in modeling academic English as he revoiced or restated his students' utterances in everyday English (Enyedy, Rubel, Castellon, Mukhopadhyay, Esmonde, & Secada, under review). Because of the ways that gesture was used as a resource to elaborate talk—and in particular the talk in English—students were able to grapple with mathematical concepts. The confusion of transferring from the English language to mathematical discourse was minimized through the use of gestural performance and speech.

Mr. Garcia's gesture and accompanying speech also served as a tool to elicit participation from students in discussing their mathematical reasoning, thoughts, and hypotheses in a whole class setting. Students' proficiency in the English language as well as their fluency of mathematical terminology was not an obstacle for them as they discussed mathematics. Once conversations were initiated, the use of gesture and speech continued to be a tool for the students as they communicated their mathematical ideas to their classmates and teacher. Teacher initiation of mathematical conversation so that students can maintain a discussion is critical for students learning mathematical concepts as well as developing their academic English.

Preliminary studies on ELLs suggest that students have difficulties solving word problems and understanding vocabulary (Cocking & Mestre, 1988; Cuevas, 1984). This focus has lent in implying and emphasizing that ELLs should first learn the appropriate vocabulary and computational skills in order to attain mathematical knowledge. The trajectory theory (teaching vocabulary acquisition first, multiple meanings second and mathematical discourse last) claims that students must first acquire vocabulary and then construct multiple meaning before they can participate in mathematical discourse, reinforcing the deficit model for ELLs. Evidence from this study supports a pedagogical focus on mathematical discourse so that ELLs can learn the lexicon, the structure of the English language, and the mathematical content as they participate and communicate with each other. For ELLs, mathematical discourse enhances listening and speaking skills in English as well as increases mathematical knowledge. We argue that with the appropriate use of gestures, accompanying talk, and/or graphical representation, an emphasis on mathematical discourse can replace the trajectory method of teaching ELLs. Gesture and speech can serve a valuable role in creating effective discourse environment for ELLs. Mathematical concepts can be discussed without emphasizing ELLs' proficiency in academic English and/or fluency in mathematical registers.

# **Contextualizing the traditional trajectory (5.1)**

Our findings demonstrate that learning terminology in mathematics is not as crucial as learning and understanding mathematics. Although students may learn or memorize mathematical terminology, there is no evidence that students will then be able to discuss or understand mathematical concepts (Dale & Cuevas, 1987; Garrison & Mora, 1999; Gutierrez, 2002; Moschkovich, 1999a; 199b; 2000; 2002a; Secada, 1992). Students need to be able to understand the mathematics in their own terms in order to bridge other mathematical concepts or relate the mathematical concepts to other academic disciplines. Our findings suggest that gesture and talk can lend in connecting everyday English with mathematic concepts and can do so without relying on academic English or mathematical registers. By reversing the order of the traditional trajectory and putting mathematical discourse first, ELLs can discuss mathematics and simultaneously learn the specialized registers as well as resolve multiple meanings.

The excerpts in this paper illustrate that discussing mathematics in a public setting enabled group discourse to be constructed without depending on the mathematical vocabulary. In Excerpt 1, the teacher was able to reference the *point of intersection* as he waved his hand in one direction and used his other hand to wave in another direction as he verbally said, "not where they cross." Mr. Garcia could have used the word *"intersection,"* but instead he chose to use everyday English<sup>12</sup>. Although specialized vocabulary of mathematics was rare in this class, students responded to their teacher's questions and discussed mathematical concepts. Gesture and talk facilitated a whole-class discourse in English without the use of mathematical registers. Using everyday meanings and/or students' first language was a resource that helped students construct their understanding of mathematics.

<sup>&</sup>lt;sup>12</sup> Whether Mr. Garcia intentionally used gesture and speech for this purpose is not known.

Another example in which the order of the trajectory was reversed was Excerpt 3, where the teacher used gesture and talk to resolve multiple meanings. Mr. Garcia used his gestures to help clarify the phrase *"mas pronto,"* which could have had various interpretations. His gestures acting out "time" and not "speed" created a visual representation for students to grapple with the specific meaning, thereby minimizing potential confusion among other students. This example demonstrated how the mapping between national languages and technical registers created opportunities for students to resolve multiple meanings that arose. Through this process, students can better understand the formal English and discourse of mathematics. It is also important to realize that students' struggles to resolve multiple meanings across national languages and technical registers require explicit attention in the discourse environment. However, these struggles are not a deficit. Instead, they can be a rich resource for meaning-making (Moschkovich, 2002).

In order for students to feel comfortable in discussing their mathematical reasoning they need to be assured that their teacher will not dismiss their mathematical rationale due to their manner of communicating. Teachers can build on students' confidence by displaying students' reasoning, hypothesis, and knowledge, as well as validating students' contributions. The validation and the broadcasting of students' mathematical contributions can build students' self-esteem in discussing mathematics. Students may then be more likely to continue communicating and discussing mathematical concepts with their peers or teacher. In this way, students' confidence in "doing" mathematics can be enhanced (McLeod, 1992).

It is important to engage students so that they can become active participants in their own learning. Students should feel and know that their mathematical contribution facilitates their learning as well as their classmates' learning. By carefully listening to students' utterances and extending their contributions to another level, teachers can empower students. This approach not only heightens students' self-esteem and positions the students as mathematical scholars, but also presents the students as knowledgeable individuals who contribute to the mathematical lesson.

# **Contextualizing the role of gestures (5.2)**

Gesture and talk are important resources in instructional conversations, especially when mathematical topics are abstract, general, or difficult to express for students learning a second national language. We argue that gesture and talk may help in the transition to and appropriation of mathematical registers and academic English. Moreover, teachers need to (1) bridge mathematics and English so that students are able to make appropriate connections when they read mathematical problems, (2) incorporate the mathematical registers and display the academic English that is necessary to understand mathematical word problems and write mathematical statements, (3) continually transition the use of gesture in the classroom to those topics that are at the leading edge of their students' conceptual or linguistic competence, and (4) emphasize formal ways of conveying meaning around topics that students already understand. These transitions are imperative for students.

It is important to note that although gesture can facilitate learning, teachers must introduce the formal register. Relying exclusively on constructing meaning using gesture and everyday language may hinder some ELLs' language development. In such a case, ELLs may depend on gesture and talk and never make an effort to learn the mathematical terminology and develop their proficiency in academic English. Without appropriating the mathematical terminology and academic English, students are at risk for being labeled as "deficient."

Given that people's perceptions of competence are often based on communicative events, students may be labeled as not knowledgeable or not proficient in a particular mathematics course because they lack the proper skills needed to engage in academic discourse, despite the fact that they understand mathematical concepts and can discuss them in informal ways. This becomes even more striking when one considers that the highest stakes evaluations of students' mathematical competence are written in the more formal, academic discourse of mathematics. In many states, exit-exams are a graduation requirement.

An emphasis on the English language is essential and critical so that students can continue developing language skills as well as learning academic content. Without emphasizing the English language and the mathematical content simultaneously, students may be in a position that impedes them when taking exit-exams, furthering their education, or seeking employment. Therefore, it is important that students bridge from informal to formal mathematical talk. Part of this transition will require the spoken word to increasingly convey more of the mathematical meanings within a conversation. In conclusion, gesture and speech can be a resource for students, but it should be a transitional resource used to engage students in meaningful discussion and should diminish as their competence in academic English ability and conceptual understanding grows.

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